Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding the Area of a Region **In Exercises 1–10, sketch the region bounded by the graphs of the equations and find the area of the region.**

1. $y = 6 - \frac{1}{2}x^2, \quad y = \frac{3}{4}x, \quad x = -2, \quad x = 2$

2. $y = \frac{1}{x^2}, \quad y = 4, \quad x = 5$

3. $y = \frac{1}{x^2 + 1}, \quad y = 0, \quad x = -1, \quad x = 1$

4. $x = y^2 - 2y, \quad x = -1, \quad y = 0$

5. $y = x, \quad y = x^3$

6. $x = y^2 + 1, \quad x = y + 3$

7. $y = e^x, \quad y = e^2, \quad x = 0$

8. $y = \csc x, \quad y = 2, \quad \frac{\pi}{6} \le x \le \frac{5\pi}{6}$

9. $y = \sin x, \quad y = \cos x, \quad \frac{\pi}{4} \le x \le \frac{5\pi}{4}$

10. $x = \cos y, \quad x = \frac{1}{2}, \quad \frac{\pi}{3} \le y \le \frac{7\pi}{3}$

Finding the Area of a Region **In Exercises 11–14, use a graphing utility to graph the region bounded by the graphs of the equations, and use the integration capabilities of the graphing utility to find the area of the region.**

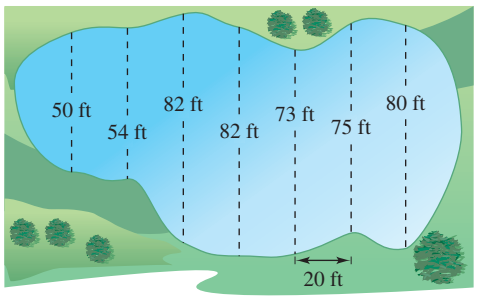
11. $y = x^2 - 8x + 3, \ y = 3 + 8x - x^2$

12. $y = x^2 - 4x + 3, \ y = x^3, \ x = 0$

13. $\sqrt{x} + \sqrt{y} = 1, \ y = 0, \ x = 0$

14. $y = x^4 - 2x^2, \ y = 2x^2$

15. Numerical Integration Estimate the surface area of the pond using (a) the Trapezoidal Rule and (b) Simpson's Rule.



16. Revenue The models $R_1 = 6.4 + 0.2t + 0.01t^2$ and $R_2 = 8.4 + 0.35t$ give the revenue (in billions of dollars) for a large corporation. Both models are estimates of the revenues from 2015 through 2020, with $t = 15$ corresponding to 2015. Which model projects the greater revenue? How much more total revenue does that model project over the six-year period?

Finding the Volume of a Solid **In Exercises 17–22, use the disk method *or* the shell method to find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given line(s).**

17. $y = x, \quad y = 0, \quad x = 3$

(a) the x-axis (b) the y-axis

(c) the line $x = 3$ (d) the line $x = 6$

18. $y = \sqrt{x}, \quad y = 2, \quad x = 0$

(a) the x-axis (b) the line $y = 2$

(c) the y-axis (d) the line $x = -1$

19. $y = \frac{1}{x^4 + 1}, \quad y = 0, \quad x = 0, \quad x = 1$

revolved about the y-axis

20. $y = \frac{1}{\sqrt{1 + x^2}}, \quad y = 0, \quad x = -1, \quad x = 1$

revolved about the x-axis

21. $y = \frac{1}{x^2}, \quad y = 0, \quad x = 2, \quad x = 5$

revolved about the y-axis

22. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1$

revolved about the x-axis

23. Depth of Gasoline in a Tank A gasoline tank is an oblate spheroid generated by revolving the region bounded by the graph of

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

about the y-axis, where x and y are measured in feet. Find the depth of the gasoline in the tank when it is filled to one-fourth its capacity.

24. Using Cross Sections Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 9$ and the cross sections perpendicular to the x-axis are equilateral triangles.

Finding Arc Length **In Exercises 25 and 26, find the arc length of the graph of the function over the indicated interval.**

25. $f(x) = \frac{4}{5}x^{5/4}, \quad [0, 4]$ **26.** $y = \frac{1}{6}x^3 + \frac{1}{2x}, \quad [1, 3]$

27. Length of a Catenary A cable of a suspension bridge forms a catenary modeled by the equation

$$y = 300 \cosh\left(\frac{x}{2000}\right) - 280, \quad -2000 \le x \le 2000$$

where x and y are measured in feet. Use the integration capabilities of a graphing utility to approximate the length of the cable.

28. Approximation Determine which value best approximates the length of the arc represented by the integral

$$\int_0^1 \sqrt{1 + \left[\frac{d}{dx}\left(\frac{4}{x+1}\right)\right]^2}\,dx.$$

(Make your selection on the basis of a sketch of the arc and *not* by performing any calculations.)

(a) 10 (b) -5 (c) 2 (d) 4 (e) 1

29. Surface Area Use integration to find the lateral surface area of a right circular cone of height 4 and radius 3.

30. Surface Area The region bounded by the graphs of $y = 2\sqrt{x}$, $y = 0$, $x = 3$, and $x = 8$ is revolved about the x-axis. Find the surface area of the solid generated.

31. Work A force of 5 pounds is needed to stretch a spring 1 inch from its natural position. Find the work done in stretching the spring from its natural length of 10 inches to a length of 15 inches.

32. Work A force of 50 pounds is needed to stretch a spring 1 inch from its natural position. Find the work done in stretching the spring from its natural length of 10 inches to double that length.

33. Work A water well has an 8-inch casing (diameter) and is 190 feet deep. The water is 25 feet from the top of the well. Determine the amount of work done in pumping the well dry, assuming that no water enters it while it is being pumped.

34. Boyle's Law A quantity of gas with an initial volume of 2 cubic feet and a pressure of 800 pounds per square foot expands to a volume of 3 cubic feet. Find the work done by the gas. Assume that the pressure is inversely proportional to the volume.

35. Work A chain 10 feet long weighs 4 pounds per foot and is hung from a platform 20 feet above the ground. How much work is required to raise the entire chain to the 20-foot level?

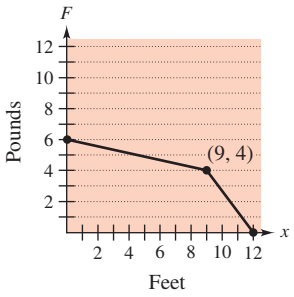
36. Work A windlass, 200 feet above ground level on the top of a building, uses a cable weighing 5 pounds per foot. Find the work done in winding up the cable when

(a) one end is at ground level.

(b) there is a 300-pound load attached to the end of the cable.

37. Work The work done by a variable force in a press is 80 foot-pounds. The press moves a distance of 4 feet, and the force is a quadratic of the form $F = ax^2$. Find a.

38. Work Find the work done by the force F shown in the figure.



39. Center of Mass of a Linear System Find the center of mass of the point masses lying on the x-axis.

$m_1 = 8, \quad m_2 = 12, \quad m_3 = 6, \quad m_4 = 14$

$x_1 = -1, \quad x_2 = 2, \quad x_3 = 5, \quad x_4 = 7$

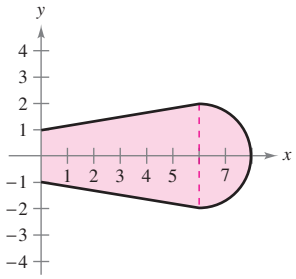
40. Center of Mass of a Two-Dimensional System Find the center of mass of the given system of point masses.

m_i	3	2	6	9
(x_i, y_i)	(2, 1)	$(-3, 2)$	$(4, -1)$	(6, 5)

Finding a Centroid **In Exercises 41 and 42, find the centroid of the region bounded by the graphs of the equations.**

41. $y = x^2, \quad y = 2x + 3$ **42.** $y = x^{2/3}, \quad y = \frac{1}{2}x$

43. Centroid A blade on an industrial fan has the configuration of a semicircle attached to a trapezoid (see figure). Find the centroid of the blade.



44. Finding Volume Use the Theorem of Pappus to find the volume of the torus formed by revolving the circle $(x - 4)^2 + y^2 = 4$ about the y-axis.

45. Fluid Force of Seawater Find the fluid force on the vertical plate submerged in seawater (see figure).

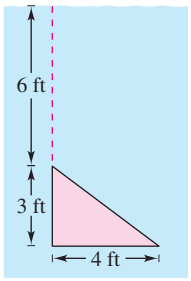


Figure for 45

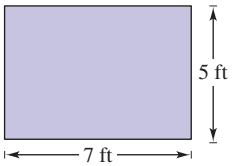


Figure for 46

46. Force on a Concrete Form The figure is the vertical side of a form for poured concrete that weighs 140.7 pounds per cubic foot. Determine the force on this part of the concrete form.

47. Fluid Force A swimming pool is 5 feet deep at one end and 10 feet deep at the other, and the bottom is an inclined plane. The length and width of the pool are 40 feet and 20 feet. If the pool is full of water, what is the fluid force on each of the vertical walls?